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COMMENT

The question of completeness and locality in Liddy's hidden variable theory

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Abstract. More details are required about the hidden variable model proposed by Liddy before it can be said to be a complete objective local hidden variable theory.

The famous inequality first proposed by Bell (1964) has initiated scores of papers (for a review see Clauser and Shimony (1978)) arguing against the possibility of explaining quantum systems behaviour by using a local hidden variable theory. While the techniques used in these arguments have been challenged (de la Pena *et al* 1972, Koc 1983), few proposals for feasible hidden variable models have been made. Two such models have been developed by Liddy, one for spin-$\frac{1}{2}$ particles (Liddy 1983) and the other to explain an experiment involving photon pair production suggested by Clauser and Horne (1975). Since this latter model (Liddy 1984) involves an experiment which has been actually performed, it will be the one examined in this paper.

Before examining a hidden variable theory, one should carefully define what is meant by the term 'hidden variables'. Generally, hidden variables are understood to be some unknown parameters whose variation in otherwise identical quantum systems explains the dispersion in measurement outcomes. Probability in quantum mechanics then takes on the same significance as in classical physics and quantum predicted expectation values are determined by averaging over the hidden variables.

Thus, for example, if A_a is the measurement result for a polariser oriented in the $\hat{a}$ direction, then the expected value of A_a, given knowledge of the hidden parameters λ, should be one of the possible measurement outcomes. For the Clauser-Horne experiment described by Liddy's model this means that

$$E(A_a|\lambda) = \pm 1 \tag{1}$$

where $E(A_a|\lambda)$ is the conditional expectation given by λ. The quantum predicted values are found by averaging over λ,

$$E(A_a)_{\rm QM} = \int E(A_a|\lambda)P(\lambda)\,d\lambda \tag{2}$$

where $P(\lambda)$ is the probability density for λ.

The Clauser-Horne experiment involves two correlated photons. If we let B_b be a measurement on the second photon in the direction $\hat{b}$ then the conditional joint expectation will be

$$E(A_aB_b|\lambda) = E(A_a|\lambda)E(B_b|\lambda) \tag{3}$$

and the conditional correlation coefficient

$$\rho(A_aB_b|\lambda) = 0. \tag{4}$$

Now, suppose that the hidden parameters can be broken into two sets

$$\lambda = \lambda_1 + \lambda_2 \tag{5}$$

where λ_1 can be determined but λ_2 cannot. Then the conditional expectation of A_a given λ_1 need not be ± 1 due to averaging over λ_2 although we must have

$$|E(A_a|\lambda_1)| \leq 1. \tag{6}$$

Also, the conditional joint expectation must now be written as

$$E(A_aB_b|\lambda_1) = E(A_a|\lambda_1)E(B_b|\lambda_1) + \mathrm{cov}(A_aB_b|\lambda_1) \tag{7}$$

since λ_1 may not now explain all the correlation between A_a and B_b measurements. Likewise, the correlation coefficient $\rho(A_aB_b|\lambda_1)$ need not be identically zero.

Such a situation seems to apply to Liddy's hidden variable model. We can identify the unit vector for photon polarisation, $\hat{p}$, with λ_1 since according to Liddy

$$E(A_a|\hat{p}) = 2(\hat{a} \circ \hat{p})^2 - 1 \tag{8}$$

and

$$\rho(A_aB_b|\hat{p}) = \frac{\pi \cos(2\theta)}{2 \sin 2\theta + (\pi - 4\theta) \cos(2\theta)} \tag{9}$$

where θ is the small angle between $\hat{a}$ and $\hat{b}$.

Is it possible to find some hidden parameter λ_2 which will maintain locality while transforming Liddy's model into a complete hidden variable theory? The work of Suppes and Zanotti (1980) argues against it. Based on some general assumptions about hidden variables, they show a necessary condition for locality is that the correlation coefficient be non-negative. This is certainly not true for Liddy's model as can be demonstrated by letting $\theta = \pi/2$ in equation (9).

Thus to accept Liddy's model as a complete objective local hidden variable theory it should be demonstrated how the model can be completed and why the arguments of Suppes and Zanotti should not be applicable to it.

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